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SUPERSONIC FLOW PAST TWO INTERSECTING AND TWO PARALLEL WINGS, (U)

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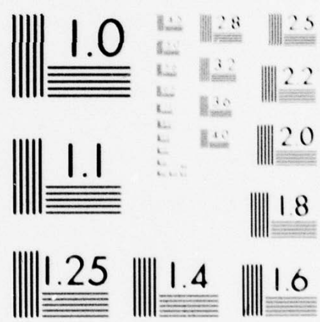

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FOREIGN TECHNOLOGY DIVISION

SUPERSONIC FLOW PAST TWO INTERSECTING
AND TWO PARALLEL WINGS

By

N. F. Vorob'yev

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

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SUPERSONIC FLOW PAST TWO INTERSECTING AND TWO PARALLEL WINGS

N. F. Vorob'yev

Here we investigate a supersonic gas flow in a dihedral angle formed by the intersecting surfaces of wings and a flow between parallel wings. The surface of the wings is slightly cambered, and at each point the tangent planes form a small angle with the velocity of the oncoming flow. Disturbances which are introduced into the flow by these surfaces are small, and it is assumed that the flow is potential and that the potential of the velocities of the disturbance satisfy the wave equation. The flow past the surfaces is studied within the framework of the theory of a thin wing, where the conditions on the surface of the wings are transferred to planes which are parallel to the velocity of the oncoming flow. For each of the wings this plane is selected such that the distances between it are small. The Volterra method of integrating the wave equation is used to solve the problem.

For intersecting wings we examine the case where the leading edges of both wings are supersonic and where the tips of the wings do not influence the region of mutual effect. For parallel wings we also obtain a solution for the case where one of the wing tips does influence the zone of mutual effect.

INTERSECTING WINGS

Motion is examined in a left-hand system of rectangular Cartesian coordinates which move with the wing. The direction of axis ox coincides with the direction of the velocity of the oncoming flow. The direction of the other axes is selected such that the plane Σ_1, onto which the boundary conditions of one wing have been transferred coincides with the plane xoz. The analogous plane of the other wing Σ_2 in this case constitutes with plane xoz the angle γ. The origin of the coordinates is selected at the point of intersection of the leading edges of the wings. Here axis ox coincides with the line which intersects the planes onto which the boundary conditions have been extended (Fig. 1).

The velocity potential is represented in the form of

$$\Phi(x, y, z) = Ux + \varphi(x, y, z).$$

The potential of velocitties φ of disturbed motion satisfy wave

equaticn

$$(M^2 - 1)\frac{\partial^2 \varphi}{\partial x^2} - \frac{\partial^2 \varphi}{\partial y^2} - \frac{\partial^2 \varphi}{\partial z^2} = 0. \qquad (1)$$

The ccnditicn cf ncnpenetiaticn cf the flcw cntc the surface of the wirgs

$$\frac{d\varphi}{dn} = -U\cos(n, x)$$

with accuracy to within small values of the seccnd order can be writter in the form of

$$\frac{d\varphi}{dN} = -U\cos(n, x), \qquad (2)$$

where the operator

$$\frac{d}{dN} = \frac{\partial}{\partial y}\cos(n, y) + \frac{\partial}{\partial z}\cos(n, z) - \frac{\partial}{\partial x}\cos(n, x)$$

is a ccnormal derivative.

The conormal derivatives on the wing $\sum_i$ $(i = 1, 2)$ with an accuracy to within small values cn the seccrd crder take the form of

$$\left.\frac{\partial\varphi}{\partial N}\right|_{\Sigma_1} = \left.\frac{\partial\varphi}{\partial y}\right|_{y=0}, \quad \left.\frac{\partial\varphi}{\partial N}\right|_{\Sigma_2} = -\frac{1-k^2}{k\sqrt{(1+k^2)}}\left.\frac{\partial\varphi}{\partial z}\right|_{y=kz}, \qquad (3)$$

where k = tg γ.

On the characteristic surface which passes through the leading edges of the wing

$$\varphi = (x, y, z) = 0 \tag{4}$$

it is convenient to introduce variables

$$x = x_1\sqrt{M^2 - 1},\ y = y_1,\ z = z_1,$$

and, in place of equation (1) for the velocity potential, obtain the following equation

$$F(\varphi) = \frac{\partial^2 \varphi}{\partial x_1^2} - \frac{\partial^2 \varphi}{\partial y_1^2} - \frac{\partial^2 \varphi}{\partial z_1^2} = 0, \tag{5}$$

whose characteristic cones have a right angle at the tip and whose conormals coincide in direction with the surfaces tangent to them.

The form in which the boundary conditions are written in the new variables is preserved. Henceforth index 1 in the variables is dropped.

Velocity potential φ at point M(x, y, z), which lies within the region of the disturbance, is determined by the Volterra formula [1,

2]

$$\varphi(x, y, z) = \frac{1}{2\pi}\frac{\partial}{\partial x}\iint\limits_{S = S_1 + S_2}\left(v\frac{\partial\varphi}{\partial N} - \varphi\frac{\partial v}{\partial N}\right)ds. \qquad (6)$$

Function v is the fundamental Volterra function of point M(x, y, z):

$$v = \log\left[\frac{(x-\xi) - \sqrt{(x-\xi)^2 - r^2}}{r}\right], \quad r = \sqrt{(y-\eta)^2 + (z-\zeta)^2}. \qquad (7)$$

The Volterra formula is obtained by means of the Green formula for the transformation of the volume integral into the surface integral for a point which does not lie on the surface carrying the Cauchy data [1]:

$$\iiint\limits_{\tau}[vF(\varphi) - \varphi F(v)]\,d\tau - \iint\limits_{S}\left(v\frac{\partial\varphi}{\partial N} - \varphi\frac{\partial v}{\partial N}\right)ds = 0, \qquad (8)$$

where F is the operator of the wave equation.

The value of the potential at point M within the region of disturbances from both wings will be, according to formula (6), determined if the values φ and $\partial\varphi/\partial N$ on the surface of the wings S are known. On the surface of the wing only the value $\partial\varphi/\partial N$ is assigned.

In the case where the angle between the plane Σ_1 and Σ_2 $\gamma = \pi/n$ (n = 1, 2, 3 ...), terms which contain function φ [1-3] can be excluded

from the right side of formula (6). In the general case ($\gamma \neq \pi/n$) terms containing φ are not excluded, and the problem reduces to solving an integro differential equation of the second type.

1. The process of eliminating function φ from the right side of formula (6) when $\gamma = \pi/n (n = 1, 2, 3 \ldots)$ is illustrated using an example where the angle between planes Σ_1 and Σ_2, to which the conditions on intersecting wings has been extended is $\gamma = \pi/2$. We select planes xoz and xoy as the planes Σ_1 and Σ_2, respectively.

The characteristic cone Γ_1 with its tip at point $M_1(x, -y, z)$, symmetrical to point $M(x, y, z)$ relative to plane Σ_1, will have total volume τ_1 with studied region τ within the dihedral angle. Volume τ_1 is bounded by the surfaces of the wings, the surface of the characteristic cone Γ_1 and part of the leading characteristic of surface σ. Within volume τ_1, into which the axis of the characteristic cone with its tip at point M_1 does not enter, the use of Gauss' (8) formula gives us the dependence

$$\iint_{S_{11}+S_{21}} \left(v_1 \frac{\partial \varphi}{\partial N} - \varphi \frac{\partial v_1}{\partial N}\right) ds = 0, \tag{9}$$

where v_1 is the fundamental function of point $M_1(x, -y, z)$. Area S_{11} coincides with area S_1, which is cut out by cone Γ on plane xoz, while S_{21} represents part of the plane xoy cut out by cone Γ_1 (Fig.

2).

Analogously for point $M_2(x, y, -z)$, which is symmetrical to point $M(x, y, z)$ relative to plane Σ_2, characteristic cone Γ_2 will have total volume τ_2 with volume τ, and the following dependence can be obtained:

$$\iint\limits_{S_{12}+S_{22}} \left(v_2 \frac{\partial\varphi}{\partial N} - \varphi \frac{\partial v_2}{\partial N}\right) ds = 0, \qquad (10)$$

where v_2 is the fundamental function of point $M_2(x, y, -z)$. Area S_{22} coincides with area S_2, which is cut out by cone Γ on plane xoy, while S_{12} represents part of the plane xoz, cut out by cone Γ_2.

For point $M_3(x, -y, -z)$, which is symmetrical to point $M_1(x, -y, z)$ in relation to plane Σ_2 and point $M_2(x, y, -z)$ relative to plane Σ_1, characteristic cone Γ_3 will have a total volume with volume τ, and the following dependence can be obtained:

$$\iint\limits_{S_{13}+S_{23}} \left(v_3 \frac{\partial\varphi}{\partial N} - \varphi \frac{\partial v_3}{\partial N}\right) ds = 0, \qquad (11)$$

where v_3 is the fundamental function of $M_3(x, -y, -z)$. Area S_{13} coincides with area S_{12}, which is cut out by cone Γ_2 on plane Σ_1, while area S_{23} coincides with area S_{21}, cut out by cone Γ_1 on plane Σ_2.

Conormal derivatives $\partial v/\partial N$ on planes $\sum_i$ $(i = 1, 2)$, parallel to axis ox, have the following form:

$$\left.\frac{\partial v}{\partial N}\right|_{\Sigma_i} = \frac{(x-\xi)(y-k_i z)}{\sqrt{1+k_i^2}\,[(y-k_i\zeta)^2+(z-\zeta)^2]\sqrt{(x-\xi)^2-[(y-k_i\zeta)^2+(z-\zeta)^2]}}, \qquad (12)$$

where $k_1 = 0$, and in the case where $\gamma = \pi/2,\ 3/2\pi$,

$$\left.\frac{\partial v}{\partial N}\right|_{\Sigma_2} = \frac{(x-\xi)z}{[(y-\eta)^2+z^2]\sqrt{(x-\xi)^2-[(y-\eta)^2+z^2]}}.$$

From formulas (7) and (12) it is apparent that when point M lies on plane $\sum_i$, then the conormal derivative of function v on this plane reverts to zero.

For point $M_i(x_i, y_i, z_i)$, which is symmetrical to point M(x, y, z) relative to plane $\sum_i$, on plane $\sum_i$ itself, according to (7) and (12) we have the relationship

$$v_i|_{\Sigma_i} = v|_{\Sigma_i}, \quad \left.\frac{\partial v_i}{\partial N}\right|_{\Sigma_i} = -\left.\frac{\partial v}{\partial N}\right|_{\Sigma_i}, \qquad (13)$$

where v_i is the fundamental Volterra function, corresponding to point $M_i(x_i, y_i, z_i)$.

On surface $\sum_1$ ($\sum_2$ when $\gamma = \pi/2$), where $\eta = 0$, $(\zeta = 0)$, according

to formulas (3) $\partial/\partial N=\partial/\partial\eta$, $(\partial/\partial N=\partial/\partial\zeta)$ and, on the strength of the properties of (13) of the fundamental function, will have

$$v_1=v,\ v_2=v_3,\ (v=v_2,\ v_1=v_3)$$
$$\frac{\partial v_1}{\partial\eta}=-\frac{\partial v}{\partial\eta},\ \frac{\partial v_2}{\partial\eta}=-\frac{\partial v_3}{\partial\eta},\ \left(\frac{\partial v_2}{\partial\zeta}=-\frac{\partial v}{\partial\zeta},\ \frac{\partial v_1}{\partial\zeta}=-\frac{\partial v_3}{\partial\zeta}\right). \qquad (14)$$

To the right side of formula (6) we add operators $\frac{1}{2\pi}\frac{\partial}{\partial x}$ of the left sides of equations (9), (10), and (11) and, considering relationship (14), we get

$$\varphi(x,y,z)=\frac{1}{\pi}\frac{\partial}{\partial x}\left\{\iint\limits_{S_1}\left[v\frac{\partial\varphi}{\partial\eta}\right]_{\eta=0}d\xi\,d\zeta+\iint\limits_{S_1}\left[v\frac{\partial\varphi}{\partial\zeta}\right]_{\zeta=0}d\xi\,d\eta+\right.$$
$$\left.+\iint\limits_{S_{II}}\left[v_2\frac{\partial\varphi}{\partial\eta}\right]_{\eta=0}d\xi\,d\zeta+\iint\limits_{S_{II}}\left[v_1\frac{\partial\varphi}{\partial\zeta}\right]_{\zeta=0}d\xi\,d\eta\right\}. \qquad (15)$$

After differentiating with respect to x over the right side of equation (15) (terms containing derivatives with respect to x from variable integration limits, all revert to zero), we get the value for the potential at point P, which lies within the dihedral angle $\gamma=\pi/2$:

$$\varphi(x,y,z)=-\frac{1}{\pi}\left\{\iint\limits_{S_1}\frac{\left.\frac{\partial\varphi}{\partial\eta}\right|_{\eta=0}d\xi\,d\zeta}{\sqrt{(x-\xi)^2-[y^2+(z-\zeta)^2]}}+\right.$$
$$+\iint\limits_{S_1}\frac{\left.\frac{\partial\varphi}{\partial\zeta}\right|_{\zeta=0}d\xi\,d\eta}{\sqrt{(x-\xi)^2-[(y-\eta)^2+z^2]}}+\iint\limits_{S_{II}}\frac{\left.\frac{\partial\varphi}{\partial\eta}\right|_{\eta=0}d\xi\,d\zeta}{\sqrt{(x-\xi)^2-[y^2+(z+\zeta)^2]}}+$$
$$\left.+\iint\limits_{S_{II}}\frac{\left.\frac{\partial\varphi}{\partial\zeta}\right|_{\zeta=0}d\xi\,d\eta}{\sqrt{(x-\xi)^2-[(y+\eta)^2+z^2]}}\right\}. \qquad (16)$$

2. The angle between planes Σ_1 and Σ_2, to which the conditions on the wings are extended, is $\gamma \neq \pi/n$.

If point M(x, y, z) lies on plane Σ_1, then the axis of the characteristic cone, where function v breaks, lies entirely within the plane Σ_1. Formula (8) can be used if we eliminate the axis of the cone [1] from region τ. In place of a volume bounded by surfaces $\Sigma+\sigma+\Gamma$, let us examine a volume bounded by surfaces $\Sigma+\sigma+\Gamma'+C_{\epsilon}$, where C_{ϵ} is the surface of half of a cylinder of radius whose axis coincides with the axis of characteristic cone Γ; Γ' - surface of half-cone with apex at point M, close to surface of characteristic cone Γ.

Take into consideration the fact that on the leading portion of the characteristic surface σ functions φ and $\partial\varphi/\partial N$, are equal to zero and on surface Σ where the axis of the characteristic cone has been eliminated, according to (12), the conormal derivative $\left.\frac{\partial v}{\partial N}\right|_{\Sigma_1}$ is equal to zero. Then, in the case of passage to the limit with subsequent differentiation with respect to x, from equation (8) we derive the formula for the velocity potential at point M, which lies on plane Σ_1:

$$\varphi|_{\Sigma_i} = \frac{1}{\pi}\frac{\partial}{\partial x}\left\{\iint\limits_{S_i} v\frac{\partial\varphi}{\partial N}ds + \iint\limits_{S_j}\left(v\frac{\partial\varphi}{\partial N} - \varphi\frac{\partial v}{\partial N}\right)ds\right\}, \tag{17}$$

where $S_i\,(S_j)$ is the part of plane $\sum_i\left(\sum_j\right)$, which is cut out by the characteristic cone Γ whose apex is at point M. Formula (17) expresses the function φ on plane $\sum_i$ in terms of known functions $\partial\varphi/\partial N$ on planes $\sum_i$ and $\sum$ and in terms of the unknown function f on plane $\sum_j$. In the general case of $\gamma \neq \pi/n$ we must simultaneously study a system of two integrodifferential equations of the second type of (17).

In the case of $0 < \gamma < \pi$, formula (17) can be converted into a form in which the right side will contain the value of the potential φ on plane $\sum_i$, while the term which contains function φ on plane $\sum_j$ is eliminated. For this we select point M_1, which is symmetrical to point $M|_{\Sigma_i}$ in relation to plane $\sum_j$. For point M_1 our plots are analogous to those obtained above in the case of $\gamma = \pi/2$ for the elimination of function φ on corresponding planes, and, considering the properties of (13) of the characteristic functions from formula (17), we get the formula

$$\varphi|_{\Sigma_i} = \frac{1}{\pi}\frac{\partial}{\partial x}\left\{\iint\limits_{S_i} v\frac{\partial\varphi}{\partial N}ds + 2\iint\limits_{S_j} v\frac{\partial\varphi}{\partial N}ds + \iint\limits_{S_{ij}} v_j\frac{\partial\varphi}{\partial N}ds - \iint\limits_{S_{ij}} \varphi\frac{\partial v_j}{\partial N}ds\right\}, \tag{18}$$

where S_{ij} is that part of the wing plane $\sum_j$, which is cut out by the

characteristic cone Γ_j, whose apex is at point M_j, while v_j is the characteristic Volterra function of point M_j.

In the first three terms of the braces in formula (18) differentiation with respect to x can be performed directly. Here terms from the differentiation with respect to the boundaries of the region, which depend on x, revert to zero. In the last term of the braces we must first integrate in parts with respects to variable ξ and then perform the operation of differentiation with respect to x.

For point M, which lies on surface Σ_1, formula (18) takes the form of

$$\varphi(x, 0, z) = -\frac{1}{\pi}\left\{\iint\limits_{S_1} \frac{\left.\frac{\partial\varphi}{\partial\eta}\right|_{\eta=0} d\xi\, d\zeta}{\sqrt{(x-\xi)^2-(z-\zeta)^2}} - \frac{2(1-k^2)}{k}\iint\limits_{S_1} \frac{\left.\frac{\partial\varphi}{\partial\zeta}\right|_{\eta=k\zeta} d\xi\, d\zeta}{\sqrt{(x-\xi)^2-[(z-\zeta)^2+k^2\zeta^2]}} + \iint\limits_{S_{11}} \frac{\left.\frac{\partial\varphi}{\partial\eta}\right|_{\eta=0} d\xi\, d\zeta}{\sqrt{(x-\xi)^2-f_1^2(z,\zeta)}} + \frac{2kz}{1+k^2}\int\limits_0^{z_1(x)}\int\limits_{\psi_1(\zeta)}^{x-f_1(z,\zeta)} \frac{\varphi'_\xi(\xi,0,\zeta)(x-\xi)\, d\xi\, d\zeta}{f_1^2(z,\zeta)\sqrt{(x-\xi)^2-f_1^2(z,\zeta)}}\right\}. \quad (19)$$

where $\xi=\psi_1(\zeta)$ is the equation of the leading edge of the wing Σ_1; $z_1(x)$ is the coordinate of the point of intersection of the leading edge of the wing Σ_1 with hyperbole $(x-\xi) - f_1(z, \zeta) = 0$, where

$$(x-\xi)-f_1(z,\zeta)=0, \text{ where } f_1(z,\zeta)=\sqrt{\left(\frac{2k}{1+k^2}z\right)^2+\left(\frac{1-k^2}{1+k^2}z-\zeta\right)^2}.$$

For point M which lies on surface Σ_2, formula (18) takes the form of

$$\varphi(x,kz,z)=-\frac{1}{\pi}\left\{\frac{k^2-1}{k}\iint_{S_2}\frac{\left.\frac{\partial\varphi}{\partial\zeta}\right|_{\eta=k\zeta}d\xi\,d\zeta}{\sqrt{(x-\xi)^2-(1+k^2)(z-\zeta)^2}}+\right.$$
$$+2\iint_{S_1}\frac{\left.\frac{\partial\varphi}{\partial\eta}\right|_{\eta=0}d\xi\,d\zeta}{\sqrt{(x-\xi)^2-[k^2z^2+(z-\zeta)^2]}}+$$
$$+\frac{k^2-1}{k}\iint_{S_{21}}\frac{\left.\frac{\partial\varphi}{\partial\zeta}\right|_{\eta=k\zeta}d\xi\,d\zeta}{\sqrt{(x-\xi)^2-f_2^2(z,\zeta)}}-$$
$$\left.-2kz\int_0^{z_2(x)}\int_{\psi_2(\zeta)}^{x-f_2(z,\zeta)}\frac{\varphi'_\xi(\xi,k\zeta)(x-\xi)\,d\xi\,d\zeta}{f_2^2(z,\zeta)\sqrt{(x-\xi)^2-f_2^2(z,\zeta)}}\right\}. \tag{20}$$

where $\xi=\psi_2(\zeta)$ is the equation for the projection of the leading edge of the wing Σ_2 onto plane xoz, and $z_2(x)$ - the coordinate of the intersection point of the leading edge of the wing Σ_2 with hyperbole $(x-\xi)-f_2(z,\zeta)=0$, where

$$f_2(z,\zeta)=\sqrt{k^2(z+\zeta^2)+(z-\zeta)^2}.$$

Equations (19 and (20) represent integrodifferential equations of the same type and can be solved by the method of successive approximations. Here, when φ_ξ is found from the preceding approximation, we must integrate in parts with respect to the variable contained in the integrand with x before going on to the

operation of differentiating with respect to x.

Formula (18), obtained for a point lying on surface Σ_i can also be conveniently used when $\gamma = \pi/n$. For exaple, in the case of $\gamma = \pi/4$ for point M_{ji}, symmetrical to point M_j in relation to plane Σ_i, and for point M_{jij}, symmetrical to point M_{ji} relative to plane Σ_j (point M_{jij} lies in plane Σ_i). our plots are anacgous to those described for the case of $\gamma = \pi/2$ for a point within the cihedral angle. Taking into account the properties of (13) of the characteristic functions and the equality to zero on plane Σ_i of cormal derivative $\left.\frac{\partial v_{ji}}{\partial N}\right|_{\Sigma_i}$, which emerges from formula (12), where v_{ji} is the characteristic Volterra function of point M_{ji}. formula (18) can be transformed into

$$\varphi|_{\Sigma_i} = -\frac{1}{\pi}\left\{\iint\limits_{S_i}\frac{\partial\varphi}{\partial N}\frac{ds}{\sqrt{(x-\xi)^2-r^2}} + 2\iint\limits_{S_j}\frac{\partial\varphi}{\partial N}\frac{ds}{\sqrt{(x-\xi)^2-r^2}} + 2\iint\limits_{S_{ij}}\frac{\partial\varphi}{\partial N}\frac{ds}{\sqrt{(x-\xi)^2-r_j^2}} + 2\iint\limits_{S_{ji}}\frac{\partial\varphi}{\partial N}\frac{ds}{\sqrt{(x-\xi)^2-r_{ji}^2}} + \iint\limits_{S_{jij}}\frac{\partial\varphi}{\partial N}\frac{ds}{\sqrt{(x-\xi)^2-r_{jij}^2}}\right\},$$

where S_i, S_j, S_{ij} are determined in formulas (17) and (18); S_{ji} (S_{jij}) - the area of plane $\Sigma_j(\Sigma_i)$. cut out by characteristic cone $\Gamma_{ji}(\Gamma_{jij})$ whose tip is at point M_{ji} (M_{jij}); here $r_k = \sqrt{(y_k-\eta)^2+(z_k-\zeta)^2}$, where index k corresponds to the index of points M, M_j, M_{ji}, M_{jij}; η, ζ represent the current coordinates on the corresponding plane on which the integration is done.

PARALLEL WNGS

Here we exaine two slightly cambered wings at small angles of attack to the velocity of the oncoming flow. The conditions on the surface of the wings, as is standard practice in the thin-wng theory, are extended to the planes Σ_1 and Σ_2, which are selected such that the velocity of the oncoming flow lies within these planes and the distances between the points on the surface of each of the wings and the corresponding plane Σ_i are small.

The distance between planes Σ_1 and Σ_2 equals h. The leading edges of both wings are supersonic, and one wing is only slightly staggered in relation to the other, so that the characteristic surface which emerges from the leading edge of one wing intersects the surface of the other wing.

As plane xoz we use the plane Σ_1, onto which the conditions on the upper wing are extended. Axis ox is directed along the flow, oy - upward, while oz is directed to the right, if one is facing the direction opposite axis ox. The study is done in a deformed system of coordinates, where the velocity potential satisfies equation (5) and the characteristic cones have a right angle at the apex.

The velocity potential in the region between the wings,

determined by the Volterra formula, can be represented in the form of

$$\varphi(x, y, z) = \frac{1}{2\pi}\frac{\partial}{\partial x}\iint\limits_{S_1+S_2}\left[v\frac{\partial\varphi}{\partial\eta} - \varphi\frac{\partial v}{\partial\eta}\right]d\xi\, d\zeta, \qquad (21)$$

were $S_1(S_2)$ is the area on the wing $\Sigma_1(\Sigma_2)$, which is cut out by cone Γ with the apex at point $M(x, y, z)$.

In the studied case of parallel wing arrangement it is possible by selecting points symmetrical to point M in relation to planes Σ_1 and Σ_2, and a corresponding selection of the fundamental functions, to eliminate from the right side of equation (21) values of function φ which are not assigned on surfaces $\sum_i$. The procedure of eliminating φ values from the right side of equation (21) depends on the number of reflections of the leading characteristic of the surface from the surface of the wings.

For point M(x, y, z) of region V_{00}, located downstream from the leading characteristic surfaces of both wings and in front of the surfaces of their first reflection (area $C_1A_1C_2E_1$ - section of region V_{00} with plane $\zeta = z$, Fig. 3), we select points M_{11} and M_{21}, which are symmetrical to point M of planes Σ_1 and Σ_2, respectively. Characteristic cone $\Gamma_1(\Gamma_2)$, whose tip is at point $M_{11}(M_{21})$, cuts out on the wing surface region $S_1(S_2)$, which coincides with region $S_1(S_2)$, which is cut out by cone Γ, whose apex is at point M on the

wing $\Sigma_1(\Sigma_2)$. The fundamental Volterra functions for points M and $M_{11}(M_{21})$ coincide on wing $\Sigma_1(\Sigma_2)$, while the conormal derivatives of the fundamental functions are different in sign (formula (13)). After performing operations analogous to those described above in the case of intersecting wings, we obtain for point M, which lies in the region V_{00}, a expression for the velocity potential in the form of

$$\varphi(x, y, z) = \frac{1}{\pi}\frac{\partial}{\partial x}\left\{\iint_{S_1}\left[v\frac{\partial\varphi}{\partial\eta}\right]_{\eta=0}\times\right.$$

$$\left.\times d\xi\,d\zeta + \iint_{S_2}\left[v\frac{\partial\varphi}{\partial\eta}\right]_{\eta=-h_1} d\xi\,d\zeta\right\}. \qquad (22)$$

For point M of region V_{10}, which lies behind the characteristic surface reflected from wing Σ_1 and before the characteristic surface reflected from wing Σ_2 (area A_1C_2A - section of region V_{10} wth plane $\zeta = z$), the procedure of selecting the symmetrical points should be continued, since in this case cone Γ_1 from point M_{11} also intersects wing Σ_2. Here, in formula (21) we add a term which contains φ in the integral with respect to area S_{21}, cut out by cone Γ_1 on wing Σ_2. In the next stage we select point M_{22}, which is symmetrical to point M_{11} in relation to wing Σ_2. On wing Σ_2 the cone which emerges from M_{22} cuts out an area equal to area S_{21}, cut out by cone Γ_1 which emerges from point M_{11}. On plane Σ_2 the fundamental Volterra functions of points M_{22} and M_{11} coincide, while the sign of their conormal derivatives is different. If we use formula (8), then from the right side of formula (21) we can eliminate terms which contain the value

of function φ and write the potential at point M, which lies within the zone of the first reflection of the leading characteristic surface from the wing Σ_1, in the form of

$$\varphi(x, y, z) = \frac{1}{\pi} \frac{\partial}{\partial x} \left\{ \iint\limits_{S_1} \left[v \frac{\partial \varphi}{\partial \eta} \right]_{\eta=0} d\xi\, d\zeta + \right.$$

$$\left. + \iint\limits_{S_2} \left[v \frac{\partial \varphi}{\partial \eta} \right]_{\eta=-h} d\xi\, d\zeta + \iint \left[v_{11} \frac{\partial \varphi}{\partial \eta} \right]_{\eta=-h_*} d\xi\, d\zeta \right\}. \qquad (23)$$

were v_{11} is the fundamental Volterra function for point M_{11}.

The process of eliminating function f from the right side of the Volterra formula for regions lying within the zone of single reflection of the leading characteristics of the surface is analogous.

Calculaton of the velocity potential by formulas (22) and (23), when the value $\partial\varphi/\partial\eta$ is known everywhere, can be done for wings of infinite span or for wings whose leading edges are entirely supersonic.

Let us examine the case where in the region of mutual wing influence the tip effect of one of the wings - Σ_1, let us say - is noticeable. Here, to shift the solution of the problem to the case of entirely supersonic edges we must know the velocity component $\partial\varphi/\partial\eta$ everywhere on plane $\eta=0$. In Fig. 4 the dashed line A'B'D'C'

designates that portion of the plane $\eta=0$, which includes wing Σ_1 itself and the zone of influence of the tips of this wing. Line A_1A_1 is the line of intersection of plane $\eta=0$ with the leading characteristic of the surface of the lower wing Σ_2. On the part of plane $\eta=0$, which lies in front of line A_1A_1 (if we are looking in the direction of the flow), the value of $\partial\varphi/\partial\eta$ is known: In region A^*CFKA_1 (σ_0) it equals zero, while in region EKH (σ_1) for the value of $\partial\varphi/\partial\eta=0$, we are familiar with the inversion formula of [4], from which σ_1 is determined outside of the wing in the case of an isolated wing (Fig. 5). In that part of plane $\eta=0$, which lies beyond line A_1A_1 outside the zone of tip influence of wing Σ_1, region A_1KTC^* (σ_2), value $\partial\varphi/\partial\eta$ is a known function, calculated by the known field of isolated wing Σ_2. In the remaining portion of plane $\eta=0$, which lies behind line A_1A_1, region KHFT (σ), value $\partial\varphi/\partial\eta=0$ is subject to definition. To determine σ in region σ we begin by writing the values of the potential in the Volterra form for point M, which lies in region σ of plane $\eta=0$. In the Volterra formula the integral from term $\varphi\frac{\partial v}{\partial\eta}$ with respect to the area cut out by the characteristic cone, whose apex is at point M on plane $\eta=0$, where point M itself lies, reverts to zero according to formula (12). In the formula for the velocity potential the integral for the area lying in plane $\eta=0$, alone remains from term $v\frac{\partial\varphi}{\partial\eta}$. The velocity potential at point M, which lies in the region σ on the upper side of plane $\eta=0$, can, according to formula (17) and (12), be represented in the form of

$$\varphi(x, 0_+, z) = \frac{1}{\pi} \frac{\partial}{\partial x} \left\{ \iint_{S_1} v \frac{\partial \varphi}{\partial \eta}\Big|_{\eta=0_+} d\xi\, d\zeta + \right.$$

$$\left. + \iint_{\sigma_1+\sigma_2} v \frac{\partial \varphi}{\partial \eta}\Big|_{\eta=0_+} d\xi\, d\zeta + \iint_{\sigma} v\theta|_{\eta=0_+} d\xi\, d\zeta \right\}. \qquad (24)$$

where S_1, σ_1, σ_2 and σ are the parts of wing Σ_1, regions $\sigma_1 \sigma_2$, and σ, respectively, described above and falling within the characteristic cone, whose apex is at point $M(x, 0, z)$. The potential at the same point M, which lies on the lower side of plane $\eta=0_-$, according to formulas (17) and (12), can be represented in the form of

$$\varphi(x, 0_-, z) = \frac{1}{\pi} \frac{\partial}{\partial x} \left\{ \iint_{S_1} v \frac{\partial \varphi}{\partial \eta}\Big|_{\eta=0_-} d\xi\, d\zeta + \iint_{\sigma_1+\sigma_2} v \frac{\partial \varphi}{\partial \eta}\Big|_{\eta=0_-} d\xi\, d\zeta + \right.$$

$$\left. + \iint_{\sigma} v\theta|_{\eta=0_-} d\xi\, d\zeta + \iint_{S_2} \left(v \frac{\partial \varphi}{\partial \eta} - \varphi \frac{\partial v}{\partial \eta} \right)_{\eta=-h_+} d\xi\, d\zeta \right\}. \qquad (25)$$

where S_2 is the region on wing Σ_2, cut out by characteristic cone Γ, whose apex is at point M. Cone Γ_2, from point $M_2(x, -2h, z)$, which is symmetrical to point $M(x, 0, z)$ relative to the plane of wing $\Sigma_2(\eta = -h_+)$, cuts out region S_2, which coincides with the region cut out by cone Γ, on wing Σ_2. Now let us look at the case where point M on wing Σ_1 is found in the interval of A_1A of a single reflection of the leading characteristics (Fig. 3) and cone Γ_2 does not intersect wing Σ_1. By using relationship (8) for volume τ_2, which is common to characteristic cones Γ and Γ_2, we transform, as above, formula (25) into the form of

$$\varphi(x, 0_-, z) = \frac{1}{\pi}\frac{\partial}{\partial x}\left\{\iint_{S_1} v\frac{\partial\varphi}{\partial\eta}\Big|_{\eta=0_-} d\xi\, d\zeta + \iint_{\sigma_1+\sigma_2} v\frac{\partial\varphi}{\partial\eta}\Big|_{\eta=0_-} d\xi\, d\zeta + \right.$$
$$\left. + \iint_{\sigma} v\theta|_{\eta=0_-}\, d\xi\, d\zeta + 2\iint_{S_2} v\frac{\partial\varphi}{\partial\eta}\Big|_{\eta=-h_2} d\xi\, d\zeta\right\}. \qquad (26)$$

Considering the fact that in region σ $\varphi(x, 0_-, z) = \varphi(x, 0_+, z)$ and $[v\theta]_{\eta=0_-} = -[v\theta]_{\eta=0_+}$, while in regions σ_1, σ_2 $\left[v\frac{\partial\varphi}{\partial\eta}\right]_{\eta=0_-} = -\left[v\frac{\partial\varphi}{\partial\eta}\right]_{\eta=0_+}$ from equations (24) and (26), after differentiating with respect to x, we get equation

$$\iint_{\sigma}\frac{\theta(\xi, 0, \zeta)\, d\xi\, d\zeta}{\sqrt{(x-\xi)^2-(z-\zeta)^2}} = F(x, z), \qquad (27)$$

$$F(x, z) = \frac{1}{2}\iint_{S_1}\frac{\left[\frac{\partial\varphi}{\partial\eta}\Big|_{\eta=0_+} - \frac{\partial\varphi}{\partial\eta}\Big|_{\eta=0_-}\right] d\xi\, d\zeta}{\sqrt{(x-\xi)^2-(z-\zeta)^2}} - \iint_{\sigma_1+\sigma_2}\frac{\frac{\partial\varphi}{\partial\eta}\Big|_{\eta=0}\, d\xi\, d\zeta}{\sqrt{(x-\xi)^2-(z-\zeta)^2}} - \iint_{S_2}\frac{\frac{\partial\varphi}{\partial\eta}\Big|_{\eta=-h_2}\, d\xi\, d\zeta}{\sqrt{(x-\xi)^2-(z-\zeta)^2}},$$

where F(x, z) is a known function.

Equation (27) represents an integral Volterra equation of the first type.

9tab If point M lies within the regon of KEL (Fig. 5), then region σ will be a right-angled isosceles triangle with the apex at point (x, z) and the base on line KH, i.e., equation (27) is the Abel equation, whose inversion is known [5]. After the value θ is determined in

region KPH, equation (27) for point M within the regions of KPNT and HEP also becomes an Abel equation [4]. After the values of θ are calculated in regions KENT and HPR, the search for the value of θ in region ENR is again reduced to solving the Abel equation.

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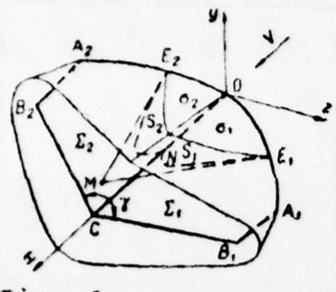

Fig. 1.

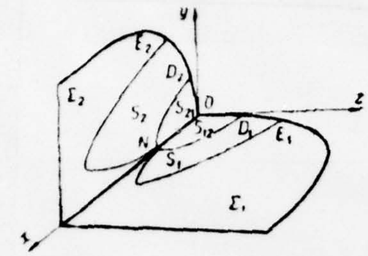

Fig. 2.

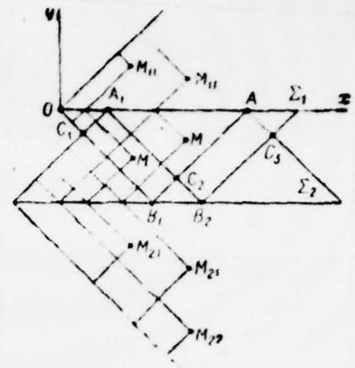

Fig. 3.

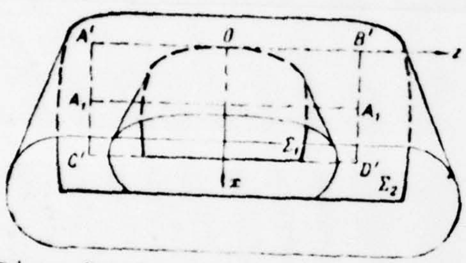

Fig. 4.

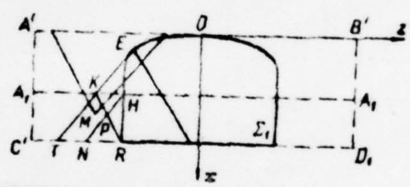

Fig. 5.

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